647510 **BSC2MatC201x** **Seat No :________**

B.Sc. Semester - 2 (*CBCS*) Examination

March/April – 2025 (New Course)

MATHEMATICS (CORE)

Time: 2.30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1 (A) Answer any two. (10)

1) Get the equation of tangent plane and normal to the sphere $x^2 + y^2 + z^2 = a^2$ at the point (α, β, γ).
2) Derive equation of a cylinder of which generator remain parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and passing through a guiding curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0\, ; z = 0.$
3) Obtain the equation of the sphere which passes through the circle $x^2 + y^2 = 4\, ; z = 0$ and cut by the plane $x + 2y + 2z = 0$ in a circle of radius 3.

Que.1(B) Answer any one. (04)

1) Show that the sphere with centre $(7,14,7)$ and radius 13 touches the plane $3x + 12y + 4z = 48$. Also find the co-ordinates of point of contact.
2) Find the equation of right circular cylinder with radius 2 and axis $\frac{x-1}{2} = y - 2 = \frac{z-3}{2}$.

Que.2 (A) Answer any two. (10)

1) State and prove Euler's theorem for homogeneous function of two variables.
2) If $u = sin^{-1}(x - y)\, ;\; x = 3t, y = 4t^3$ then show that: $\frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}$.
3) If $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)\, ; x \neq y$ then show that: $$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4sin^2 u)sin2u.$$

Que.2(B) Answer any one. (04)

1) If $f(x,y) = xy\left(\frac{x^2-y^2}{x^2+y^2}\right)\, ;\; (x,y) \neq (0,0)$

   $= 0\, ;\; (x,y) = (0,0)$

   then show that f is continuous at $(0,0)$.
2) Find $\frac{d^3y}{dx^2}$; if $ax^2 + 2hxy + by^2 = 1$.

Que.3(A) Answer any two. (10)

1) Expand $e^x cosy$ in powers of x and y up to three degrees.
2) Find volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
3) If da, db and dc are errors in measurement in a, b and c respectively as length of sides of ΔABC then prove that: $\frac{da}{cosA} + \frac{db}{cosB} + \frac{dc}{cosC} = 0$.

Que.3(B) Answer any one. (04)

1) If $f(x,y) = x^3 + xy + y^3$ then find the approximate value of $f(1.01, 2.98)$.
2) If $u = x^2 - 2y,\; v = x + y + z,\; w = x - 2y + 3z$ then show that: $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 10x + 4$.

Que.4(A) Answer any two. (10)

1) Define: Transpose of a matrix. If A and B are conformable matrices for multiplication then prove that: $(AB)^T = B^T A^T$.
2) Define: Normal form a matrix. Find the rank of matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ by representing it to normal form.
3) If A and B are square matrices of order n then prove that :
$$(i) adj(AB) = adjB \cdot adjA \quad (ii) (adjA)^T = adj(A^T).$$

Que.4(B) Answer any one. (04)

1) Define : Orthogonal matrix. If $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal then find the value of l, m and n.
2) Define: Involutary matrix. Prove that: A is involutary matrix iff $(I + A)(I - A) = 0$.

Que.5(A) Answer any two. (10)

1) State and prove Cayley-Hamilton theorem.
2) Find eigen value and eigen vectors of matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.
3) Solve: $5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5$.

Que.5(B) Answer any one. (04)

1) Show that the matrices A and A^T have the same eigen values.
2) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then express $2A^5 - 3A^4 + A^2 - 4I$ as a linear polynomial.

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